Online Appendix to

Quality Disclosure and Product Selection

Tianle Song

June, 2022

A Model with State-dependent Transfers $\{T^r\}$

In this section, we consider a model in which the incumbent's transfers $\{T^r\}$ are state-dependent. Specifically, we assume that the transfers $\{T^r\}$ are payable if entry does not occur. All other assumptions are the same as before.

First, we show that there always exists an equilibrium with no exclusion. By no exclusion, we mean an equilibrium in which all entrant types enter the market. An important point to mention here is that the existence of such an equilibrium is independent of whether the entrant discloses or not.

Lemma 1': *An equilibrium with no exclusion exists.*

Proof: Notice that accepting the entrant's offer is in a retailer's best response set if another retailer chooses to accept the entrant's offer. Therefore, if at least two retailers choose to accept the entrant's offer, an equilibrium with no exclusion exists. $\square$

In Lemma 2', we show that a successful entrant of any type will enter the market at no cost in any equilibrium with entry.

Lemma 2': *If the entrant enters the market in equilibrium, she offers a zero transfer to every retailer.*

Proof: By the proof of Lemma 1', it is optimal for the entrant to offer a zero transfer to every retailer if at least two retailers accept the entrant's offer. Now suppose that only one retailer, say retailer r, chooses to accept the entrant's offer and $T_e^r > 0$. Then, the entrant must offer a zero transfer to every other retailer in order to minimize expenditure. However, $T_e^r > 0$ cannot be supported in equilibrium, because the entrant can make herself better off by offering a slight positive transfer to another retailer r' to induce accommodation and $T_e^r = 0$ to retailer r. Therefore, if the entrant enters the market in equilibrium, she must offer a zero transfer to every retailer. $\square$

We now characterize the equilibria under complete information and under voluntary disclosure, respectively. As in the main analysis, we focus on equilibria in which an entrant is excluded whenever possible. We find that most of the results in the basic model still hold. In particular, Lemma 3 and Proposition 2 are true here, so that in every equilibrium with disclosure only an entrant with quality $q_e \in (q_e^*, 1]$ will disclose. However, by Lemma 2' the equilibrium transfers are slightly different from those in Propositions 1 and 3. We state the new results below.

Proposition 1': *There exists a unique $q_e^* \in (0,1)$ such that in each subgame perfect equilibrium*

(i) *if $q_e \in [0, q_e^*]$, then the incumbent offers $\hat{T}^*(q_e) = \pi_e(q_e)$ to every retailer and the entrant offers $\hat{T}_e^{r*}(q_e) \in [0, \pi_e(q_e)]$ to each retailer r; all retailers accept the incumbent's offer and entry is deterred;*

(ii) *if $q_e \in (q_e^*, 1]$, then the entrant offers a zero transfer to every retailer and entry is accommodated. The incumbent offers a zero transfer to every retailer if only one* *retailer accepts the entrant's offer; he offers an arbitrary transfer to each retailer if at least two retailers accept the entrant's offer.*

Proof: The proof of part (i) is similar to the proof of Proposition 1(i). Part (ii) follows from the proof of Lemma 2'. □

Proposition 2': *Consider an arbitrary equilibrium with disclosure. In equilibrium,*

(i) *an entrant with quality $q_e \in (q_e^*, 1]$ discloses and both the incumbent and the entrant offer a zero transfer to every retailer; at least one retailer accepts the entrant's offer and entry is accommodated;*

(ii) *when disclosure does not occur, the incumbent's optimal transfer to every retailer is $\hat{T}^{**} = \pi_e(\hat{q}_e^{**})$, where $\hat{q}_e^{**} \in (0, q_e^*)$;*

(iii) *an entrant with quality $q_e \in (\hat{q}_e^{**}, q_e^*]$ does not disclose and offers a zero transfer to every retailer; at least two retailers accept the entrant's offer and entry is accommodated;*

(iv) *an entrant with quality $q_e \in [0, \hat{q}_e^{**}]$ does not disclose and offers $\hat{T}_e^{r**} \in [0, \pi_e(\hat{q}_e^{**})]$ to each retailer r; all retailers accept the incumbent's offer and entry is deterred.*

Proof: The proofs of part (i), (ii) and (iv) are similar to the proofs of Proposition 3(i), 3(ii) and 3(iv). Part (iii) follows from the proof of Lemma 2'. □

Finally, we turn to the parametric conditions for an equilibrium with disclosure to exist. In this model, we have a stronger result than Proposition 4: there always exists an equilibrium with disclosure. The intuition is that since the incumbent does not need to pay the transfers when entry occurs, he always has an incentive to offer positive transfers to retailers. Then in certain cases, this incentive to transfer will drive a high-quality entrant to disclose information.

Proposition 3': *An equilibrium with disclosure exists.*

Proof: Consider the cases in which the incumbent offers $T = \pi_e(q_e^0)$ to every retailer and each retailer chooses to accept the incumbent's offer when T is weakly higher than the entrant's offer. Then an entrant with quality weakly below q_e^0 will be deterred. Following the notations in Section 3.2, the incumbent's expected profit from offering T to every retailer is

$$(1') \qquad \hat{\Pi}(T) = G^*(q_e^0)[\pi^M - nT] + [1 - G^*(q_e^0)]\left[\frac{\int_{q_e^0}^{q_e^*} \pi(q_e)dG^*(q_e)}{1 - G^*(q_e^0)} - 0\right]$$

$$= G^*(q_e^0)[\pi^M - nT] + \int_{q_e^0}^{q_e^*} \pi(q_e)dG^*(q_e),$$

where $G^*(q_e) = \frac{G(q_e)}{G(q_e^*)}$ and $q_e^0 = \pi_e^{-1}(T)$ is a strictly increasing function of T. Differentiating $(1')$ with respect to T and evaluating at $T = 0$, we have

$$(2') \qquad \hat{\Pi}'(0) = \frac{g(0)[\pi^M - \pi(0)](\pi_e^{-1})'(0)}{G(q_e^*)} > 0,$$

which implies that there always exists an optimal transfer $\hat{T}^{**} \in (0, \pi_e(q_e^*)]$ such that $\hat{\Pi}(\hat{T}^{**}) > \hat{\Pi}(0)$. Then by Lemma 3, an equilibrium with disclosure exists. □